Algebra 2 – Unit 7 Learning Guide – Exponentials & Logarithms

Lesson 1: Defining Exponential Expressions & Roots (Exponents Review) (F.IF.8b) – Feb 21/22
________ Guided Notes – Defining Exponential Expressions & Roots / Video (10 pts)
Khan Academy:
________ Exponent Rules (10 pts)
________ Negative Exponents (10 pts)
________ Fractional Exponents (10 pts)

Lesson 2: Exponential Functions (F.IF.7e, F.IF.8b) – Feb 25/26
________ Guided Notes – Exponential Functions / Video (10 pts)
Manga High:
________ Recognize the Graphs of Common Curves (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)
________ Define Exponential Functions (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)
________ Graphs and Types of Exponential Functions (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)

Lesson 3: Tables of Exponential Functions (A.CED.1) – Feb 27/28
________ Guided Notes – Tables of Exponential Functions / Video (10 pts)

Lesson 4: Defining & Evaluating Logarithms (F.LE.4) – Mar 1/4
________ Guided Notes – Defining and Evaluating Logarithms / Video (10 pts)
Khan Academy:
________ Simplifying Logarithms
________ Simplifying Logarithms 2

Lesson 5: Laws of Logarithms (F.LE.4) – Mar 5/6
________ Guided Notes – Laws of Logarithms / Video (10 pts)
Khan Academy:
________Operations with Logarithms

Lesson 6: Solving Exponential and Logarithmic Equations (F.LE.4) – Mar 7/8 and Mar 11/12
________ Guided Notes – Solving Exponential and Logarithmic Equations / Video (10 pts)
Manga High:
________ Solve Exponential Equations (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)

Lesson 7: Applications of Exponentials and Logarithms (F.LE.4) – Mar 13/14
________ Guided Notes – Applications of Exponentials and Logarithms / Video (10 pts)
Manga High:
________ Solving Problems with Exponentials (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)
________ Solving Problems with Logarithms (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)

Lesson 8: Review – Mar 25/26
________ Review (10 pts)

Lesson 9: Test – Mar 27/28
________ Test (100 pts) – Feb 20th

Khan Academy Goal: (6 components)

_______ Exponent Rules (10 pts)
_______ Negative Exponents (10 pts)
_______ Fractional Exponents (10 pts)
_______ Simplifying Logarithms
_______ Simplifying Logarithms 2
_______Operations with Logarithms

Rules/Suggestions for Khan Academy: USER____________________ Password:_________________
All assignment grades become final the day of the test. You have until that day to work on your grades. Sometime after you master (blue line / 100%) an exercise, I will enter it into the gradebook (it might take a couple of days).

If, by the day of the test, you have not mastered an exercise, you will receive the higher of either the last 10 problems worked or your progress percentage as your final score. You must complete at least one round (8 problems) to get any points for an exercise.

Your best bet with Khan Academy is to try your hardest to **get through the first 8 problems without any mistakes.** Unbeknownst to you, Khan does time you. It seems to get more stringent the longer it takes you to master something. Your first time through you can master if you just get them all correct, no matter how long it takes you (this is what we've noticed in other classes). If you go through many problems making mistakes before you finally understand what you are doing, Khan seems to require you to truly master the calculations quickly – hence my above caveat on your grades if you fail to master an exercise.

Khan does seem to have mercy on one error the second time through. Also, if you notice your mastery percentage is in the 90%+ range, let me know. You may not have to go through an entire 8-problem set to get a "complete" or 100% on your exercise.

Manga High Challenges: (6 challenges)

All assignment grades become final the day of the test. You have until that day to work on your challenges. Make sure to log in via the Winfield High School URL to ensure your score is properly recorded. *http://schools.mangahigh.com/winfieldhighschool*

USER:________________ Password:_________________

_______ Recognize the Graphs of Common Curves (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)
_______ Define Exponential Functions (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)
_______ Graphs and Types of Exponential Functions (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)
_______ Solve Exponential Equations (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)
_______ Solving Problems with Exponentials (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)
_______ Solving Problems with Logarithms (10 pts = Bronze, 15 pts = Silver, 20 pts = Gold)

Edmodo: USER:________________ Password:____________________

Guided Notes – Defining Exponential Expressions and Roots

Recall the Quotient Rule:

-

What happens if m < n?
Apply the Quotient Rule and the Definition of an Exponent:

$\frac{x^5}{x^6} =$ $\frac{x^5}{x^6} =$

So, we see that ____________exponents are actually ____________ in the denominator.

Negative Exponents: $a^{-n} = \frac{}{}$ ——

Bottom line for negative exponents:

- If the ____________ exponent is in the ____________, move the ______________ to the denominator and make the exponent ______________.
- If the ____________ exponent is in the ____________, move the ______________ to the numerator and make the exponent ______________.

Practice:

1) 2^{-5} 2) $(-2)^{-5}$ 3) -9^{-2} 4) $\frac{2^{-3}}{3^{-2}}$

Remember, we can apply all the rules of exponents even when dealing with negative exponents.

Practice:

5) $b^{-3}b^5$ 6) $-3x^{-3} \cdot 5x^2$ 7) $\frac{m^{-6}}{m^{-2}}$ 8) $\frac{4x^{-6}y^5}{-12x^{-6}y^{-3}}$

Rational Exponents

What do $\sqrt{3^2}$ and $(3^2)^{\frac{1}{2}}$ have in common?

Definition of $a^{m/n}$: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

- In fact, with every rational exponent: $a^{m/n}$,
  - "a" is the ____________
  - "m" is the ____________
  - "n" is the ____________

Name:______________________________________ Block:_____________

Practice: Write each expression using exponent or square root notation.

1) $\sqrt[4]{35}$ 2) $\sqrt[4]{xy}$ 3) $5^{\frac{1}{2}}$ 4) $a^{\frac{1}{5}}$

5) $\sqrt[3]{x^2}$ 6) $\frac{1}{\sqrt[4]{m^3}}$ 7) $5^{\frac{2}{3}}$ 8) $a^{-\frac{2}{5}}$

*** End of Guided Notes for Lesson 1 ****

Guided Notes - Exponential Functions

Consider the function f(x) = b^x or y = b^x. We have a base and an exponent still. Only this time, the exponent is a variable.

- These are called exponential functions.
- There are some limits to what values b can take and still be exponential.
  - "b" must be _____________ than ____________, but not equal to _______.
  - "x" can be ________ ___________ ______________.

Examples of Exponential Functions:

Examples of Exponential Function IMPOSTORS!!

Find the outputs and graph the functions:

$f(x) = 2^x$

Input	Output
-2	
-1	
0	
1	
2	

$g(x) = 3^x$

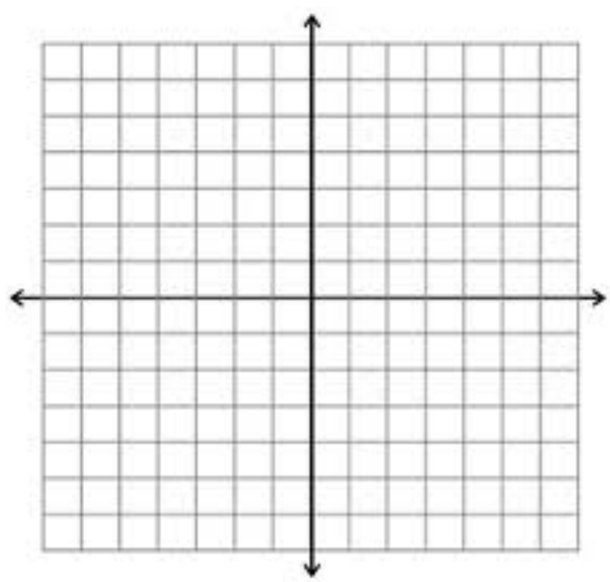
Input	Output
-2	
-1	
0	
1	
2	

$h(x) = \left(\frac{1}{2}\right)^x$

Input	Output
-2	
-1	
0	
1	
2	

$j(x) = \left(\frac{1}{3}\right)^x$

Input	Output
-2	
-1	
0	
1	
2	



Examine the differences between the outputs of your two tables. What do you see?

Characteristics of the Exponential Function:
Domain: Range:

Value of Base and its effect on direction of the function:

The Exponential Function and Transformations: Recall… **g(x) = a f(b(x – c) + d**

"a" controls vertical stretch/compression. If "a" is ____ than _____, we see stretch. If "a" is ____ than ____ and _____ than ____, we see compression.

Examples:

"b" controls horizontal stretch/compression. If "b" is ____ than _____, we see compression. If "b" is ____ than ____ and _____ than ____, we see stretch.

Examples:

"c" controls horizontal translation. If "c" is ______ than _______, we see movement to the right. If "c" is ______ than _______, we see movement to the left.

Examples:

"d" controls vertical translation. If "d" is ______ than _______, we see movement up. If "d" is ______ than _______, we see movement down.

Examples:

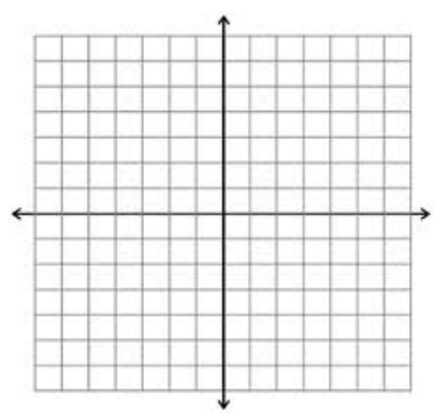
The Natural Base "e"

"e" is an __________________ number approximately equal to _____________________. "e" can be the base of an exponential function, f(x) = e^x, called the __________________ exponential function. Finding values of "e" to various powers can be done using an "e^x" key on a scientific calculator. "e" is found from the basic function:

$$f(x) = \left(1 + \frac{1}{n}\right)^n$$ (You don't need to memorize this)

$f(x) = e^x$

Input	Output
-2	
-1	
0	
1	
2	



*** End of Guided Notes for Lesson 2 ****

Guided Notes - Tables of Exponential Functions

Last time we defined a basic exponential function, but we discovered that exponential functions take other forms as well. Let's give you the FULL definition for the exponential function.

Definition: an exponential function is a function where $f(x) = a \cdot b^x$, where a_____0, b_____0 and b_____1.

We also discovered that if b_____1, the graph of the function will ___________________.

If b_____0 and b_____1, the graph of the function will___________________.

Today we will look at tables of functions. These are a different way to write functions that can be easier.

Remember linear functions with input/output tables. What did the difference column correspond to?

Let's look at some input/output tables for exponential functions:

1) $f(x) = 3^x$

Input	Output
0	
1	
2	
3	
4	

2) $f(x) = 3 \cdot 2^x$

Input	Output
0	
1	
2	
3	
4	

3) $f(x) = \frac{1}{2} \cdot 4^x$

Input	Output
0	
1	
2	
3	
4	

4) $f(x) = 6 \cdot \left(\frac{1}{3}\right)^x$

Input	Output
0	
1	
2	
3	
4	

What pattern do you notice from one output to the next in each table?

With linear functions, we had a ________ column. With exponential functions we need to have a column for ________________. This________________ corresponds to _____________.

What patterns do you notice about the value of "a" and the outputs for the various functions?

So, the value of "a" is determined by the value of the input for _________.

Practice: Determine the functions for each table.

5)

Input	Output
0	1
1	3
2	9
3	27
4	81

6)

Input	Output
0	¼
1	1
2	4
3	16
4	64

7)

Input	Output
0	3/2
1	¼
2	1/24
3	1/144
4	1/864

8)

Input	Output
0	3/5
1	6/5
2	12/5
3	24/5
4	48/5

Determining an Exponential Equation from two points: (Example on page 439 of text.)
Step 1: Determine your two points.
Step 2: Write an equation $f(x) = a \cdot b^x$ for each point.
Step 3: Divide the two equations.
Step 4: Solve for "b".
Step 5: Solve for "a"

9) (1, 6) and (3, 2/3)

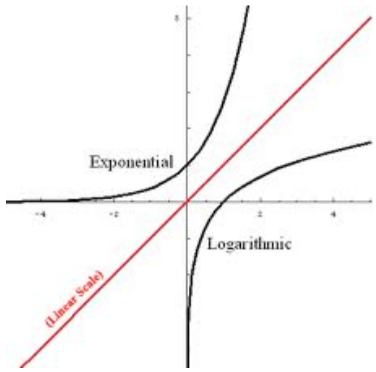
10) (2, 12) and (4, 432)

*** End of Guided Notes for Lesson 3 ****

<u>Guided Notes – Defining and Evaluating Logarithms</u>

What is a Logarithm?

- Logarithms were introduced by John Napier in the early 17th century as a means to simplify calculations.
- They were rapidly adopted by scientists, engineers, and others to perform computations more easily, using slide rules and logarithm tables.
- The logarithmic function is the inverse of the exponential function



The Exponential... $y = b^x$
Its Inverse: $x = b^y$ or $y = \log_b x$

It is read as: "y equals the log base *b* of x"

Parts of a Logarithm: $\log_2 25 = 2$ ← _______________

_______________ _______________

Expressing Logarithms in Exponential Form:

1) $\log_5 25 = 2$ 2) $\log_3 27 = 3$ 3) $\log_4 64 = 3$ 4) $\log_2 32 = 5$

Expressing Exponentials in Logarithmic Form:

5) $2^3 = 8$ 6) $5^3 = 125$ 7) $4^4 = 256$ 8) $5^4 = 625$

Evaluating Logarithms:

9) $x = \log_3 27$ 10) $x = \log_2 \sqrt{2}$ 11) $x = \log_6 \frac{1}{36}$ 12) $x = \log_2 \frac{1}{32}$

Common Logarithms: Logarithms with base 10 are called ____________________ __________________________

Common Logs are written without indicating their base – there is an **understood** base of 10
(Instead of the "disappearing 1", we have a "disappearing 10"

13) $\log 10$ 14) $\log 100$ 15) $\log 1000$

Natural Logarithms: Logarithms with base e are called ____________________ __________________________

Natural logarithms are written as:

You can rewrite a natural logarithm to be in standard logarithm form, if you desire. Most people do not.

Example: $ln\, 5$ is the same as $\log_e 5$

*** End of Guided Notes for Lesson 4 ****

Guided Notes – Laws of Logarithms

Because logarithms represent exponents, it is helpful to review laws of exponents before exploring laws of logarithms.

When _______________ like bases, __________ the_____________. $a^b a^c = a^{b+c}$

When _______________ like bases, _______________ the exponents. $\frac{a^b}{a^c} = a^{b-c}$

Product and Quotient Laws of Logarithms/Natural Logarithms:
Product Law: For all b, c > 0, $\log(bc) = \log b + \log c$ AND $\ln(bc) = \ln b + \ln c$

For all $v, w > 0$,

Quotient Law: $\log\left(\frac{v}{w}\right) = \log v - \log w$

$\ln\left(\frac{v}{w}\right) = \ln v - \ln w$

Using Product and Quotient Laws:

1. Given that log 3 = 0.4771 and log 4 = 0.6021, find log 12.

2. Given that log 40 = 1.6021 and log 8 = 0.9031, find log 5.

Power Law of Logarithms/Natural Logarithms:
For all k and v > 0, $\log v^k = k \log v$ AND $\ln v^k = k \ln v$

Example: log 9 =

Using the Power Law:

1. Given that log 25 = 1.3979, find log $\sqrt[4]{25}$

2) Given that ln 22 = 3.0910, find ln $\sqrt{22}$.

Simplifying Expressions:

1. $\log 8x + 3 \log x - \log 2x^2$

2) $\ln\left(\frac{\sqrt{x}}{x}\right) + \ln\left(\sqrt[4]{ex^2}\right)$

*** End of Guided Notes for Lesson 5 ****

Guided Notes: Solving Exponential and Logarithmic Equations

Now we can move to solve exponential equations. Remember that exponential and logarithmic functions are ___________________ functions and have the ability to "__________" each other.

This ability to "undo" means that when we have an _______________________ equation, we can take the logarithm of each side to set the exponents equal to each other.

Example:

$3^x = 3^6$

So, if the ___________ are the same, then you can set the ________________________ equal to each other and _____________ for the _____________________.

Practice:

$2^{x-3} = 2^5$ $e^{2x} = e^4$ $5^{3x-2} = 5^4$ $6^{\frac{x}{2}} = 6^4$

Sometimes we have to convert bases on one side, the other, or both to ensure both the bases are the same. Sometimes we have to factor. Sometimes we have to convert square roots to fractional exponents. Here is where it ALL comes together!

Example:

$3^x = 9^3$

Practice:

$2^x = 8^2$ $3^{2x-1} = 81^5$ $16^{4x} = 2^{32}$ $25^3 = 125^{2x-4}$

$3^x = \frac{1}{27}$ $\left(\frac{1}{8}\right)^{2x} = 4^{12}$ $2^{x^2-3x} = 2^4$ $8^{-2x} = \sqrt[4]{4}$

Now let's progress to the reverse of solving exponential functions and solve logarithmic functions. When we are dealing with logarithmic equations, we can apply the exponential function with the same base to both sides to solve for the variable.

Consider the following equation: $\log_3 x = \log_3 5$ What must x equal?

Try these: $\log_4 2x = \log_4(3x + 5)$ $\log_5 x^2 = \log_5(5x - 6)$

For this, we need to use our Laws of Logs: $2\log_4 x = \log_4 -4 + \log_4(x + 1)$

Remember that the logarithmic function is the inverse of the exponential function.
Therefore, by definition:

$$\log_b b^x = 1 \quad and \quad b^{\log_b x} = x$$

Example:

$\log_4 4^x =$ $3^{(\log_3 x)} =$

Remember that a natural logarithm is just the log, base "*e*", of something.
This means that: $\ln e^x = x$ *and* $e^{\ln x} = x$

*** End of Guided Notes for Lesson 5 ****

Guided Notes – Applications of Exponents and Logarithms

Exponential Growth/Decay Function: $\boldsymbol{f(t) = A = A_0 e^{kt}}$. If k < 0, then decay. If k > 0, then growth.

Method for Solving Growth/Decay Problems:

1) Each problem will give two sets of data. The first is a data set for a general problem. The second is a data set for a specific problem. Figure out which data belongs to which set.
    a. General data will be missing "*k*"
    b. Specific data will give information on a specific instance related to the point of the question.
2) Start with the exponential growth/decay function. Substitute in your general data and solve for *k* by taking the ln of each side.
3) With the value of *k* known, use the exponential growth/decay function and *k* to create a specific equation modeling the situation in the problem and solve for the remaining unknown.

Example (Growth): (http://censusviewer.com/city/KS/Winfield)

In the 2000 census, the number of school-aged children in Winfield, KS was 2,007. In 2010, there were 2,258 school-aged (5-17) children in Winfield, KS. In what year will the number of school-aged children in Winfield, KS reach 2,500?

Example (Decay):
Carbon-14 has a half-life of 5,715 years. That means that if a sample of a certain amount of Carbon-14 is weighed after 5,568 years, there will be half of it left. Suppose a bone purported to be from the body of King Richard III of England (b.1452, d.1485) was found under a parking lot in England. When tested that bone contained 85% of its original amount of carbon-14. What is the approximate age of this bone?

Exponent/Radical Rules Chart – Save for Use on Test and Final Exam!!!!

Name	**Exponents**	**Radicals**
Product Rule	$a^b + a^c = a^{b+c}$	$b\sqrt[n]{a} + c\sqrt[n]{a} = (b+c)\sqrt[n]{a}$
Power Rule	$(a^b)^c = a^{bc}$	$\sqrt[n]{a^b} = a^{\frac{b}{n}} = \left(a^{\frac{1}{n}}\right)^b = (a^b)^{\frac{1}{n}}$
Quotient Rule	$\frac{a^b}{a^c} = a^{b-c}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
Power of a Product Rule	$(ab)^c = a^c b^c$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Power of a Quotient Rule	$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$	
Negative Exponents	$a^{-b} = \frac{1}{a^b}$ and $\frac{1}{a^{-b}} = a^b$	
Laws of Logarithms	**Exponents**	**Logarithms (Applies to ln as well)**
Definition	$a^b = x \quad therefore, \log_a x = b$	
Identities	$b^0 = 1$ $b^1 = b$	$\log_b 1 = 0$ $\log_b b = 1$
Product Rule	$a^b a^c = a^{b+c}$	$\log_a bc = \log_a b + \log_a c$
Power Rule	$(a^b)^c = a^{bc}$	$\log_a (b)^c = \text{c} \log_a b$
Quotient Rule	$\frac{a^b}{a^c} = a^{b-c}$	$\log_a bc = \log_a b - \log_a c$
Fractional Exponents/Roots	$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$	

Regular, Common and Natural Logarithm Properties/Formulas

Regular	**Common**	**Natural**
$\log_b 1 = 0$	$\log 1 = 0$	$\ln 1 = 0$
$\log_b b = 1$	$\log 10 = 1$	$\ln e = 1$
$\log_b b^x = x$	$\log 10^x = x$	$\ln e^x = x$
$b^{\log_b x} = x$	$10^{\log x} = x$	$e^{\ln x} = x$
Change of Base Formulas		
$\log_b M = \frac{\log_a M}{\log_a b}$	$\log_b M = \frac{\log M}{\log b}$	$\log_b M = \frac{\ln M}{\ln b}$